ASTROCHALLENGE 2016 DATA RESPONSE QUESTIONS

JUNIOR ROUND

INSTRUCTIONS

- This paper consists of 14 printed pages, excluding this cover page.
- Do **NOT** turn over this page until instructed to do so.
- You have 2 hours to attempt all questions in this paper.
- At the end of the paper, submit this booklet together with your answer script.
- Your answer script should clearly indicate your school (and team number) on **EVERY** page, as well as the individuals in the said team on the first page.
- It is your team's responsibility to ensure that all pages of your answer script have been submitted, including pages to be detached from this booklet.

DRQ 1: The Galaxy Cake [20 marks]

(Disclaimer: Any resemblance of the following fictional galaxy to Messier 100/ NGC 4321 in Coma Berenices is NOT coincidental but based on actual observed data)

Once upon a time, there were two young Scientists: Carrie Alpha, and Ken Epsilon. They discovered a new galaxy simultaneously. As they were both unable to decide on a name, they decided to temporarily name it Cα-Kε, after their initials. To decide who will have the honour of deciding on the permanent name of the newfound galaxy, they both decided to challenge each other to calculate the distance of the newfound galaxy from Earth.

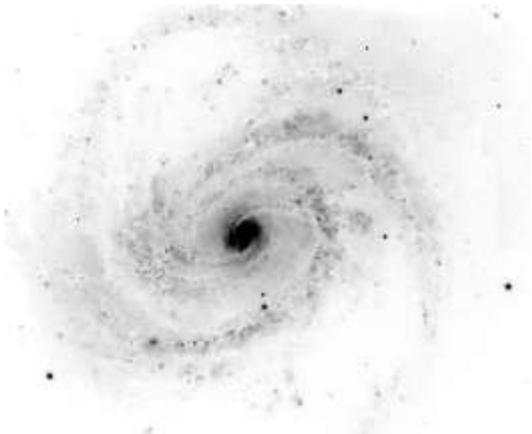


Figure 1.1 *Diagram showing newly discovered galaxy, temporarily designated Cα-Kε. Standard candles included. Actual candle brightness varies depending on type and period.*

Ken devoted 3 years to observing Type 1a Supernovas within Cα-Kε itself. He was fortunate enough to observe a total of 3 events consecutively year after year, and he jotted down the apparent magnitude of each supernova as precisely as he could from his location on an Earth-based laboratory. He was confident that because of the way Type 1a Supernovas worked, he'll obtain a reasonable estimate for the distance to Cα-Kε.

Unfortunately, his colleague, Igor Nance, has many gross misconceptions about the way Type 1a supernova works. Here is an excerpt of his writing:

> *"Type 1a supernovas usually occur between binary stars, of which one of the stars already became a white dwarf and the other star is a massive Supergiant. The massive Supergiant will engulf the white dwarf as it slowly spirals into its companion. When this happens, the white dwarf breaks up once it crosses the Chandrasekhar Limit, without electron degeneracy pressure to maintain its orbit. It then fuses with the supergiant and causes a giant supernova, leaving only a black hole in the aftermath."*

<u>Question I</u>

Help Ken write a short paragraph to his colleague on the conceptual mistakes in this excerpt. [3 marks]

Observed supernova	Date of observation	Location	Apparent Magnitude
SN 2014D*	14/02/14**	Galactic Nucleus	15.6
SN 2015R*	25/12/15**	Near Galactic Bulge	15.7
SN 2016Q*	31/10/16**	Galaxy Arm	15.3

*Disclaimer 1: Real designations are SN1901B, SN1914A and SN2006X found on those years.
**Disclaimer 2: Do not expect to always observe supernovae on Valentine's Day, Christmas or Halloween.

Question II

(a) Estimate the distance from Earth to Cα-Kε, using only information from the above given data. Provide your answer in parsecs. [3 marks]

(b) Comment on two possible sources of error which could have affected Ken's estimate of the distance between Earth and Cα-Kε, using Type 1a supernovae. [2 marks]

The Chandrasekhar limit is defined as the maximum mass of a white dwarf before it undergoes supernova. It is given by the following simplified equation for stars in the galaxy Cα-Kε, based on empirical data:

$$M_{limit} = \frac{2.02\sqrt{3\pi}}{2}\left(\frac{\hbar c}{G}\right)^{1.5}(1.525\ m_H)^{-2}$$

Whereby:
ħ is the Reduced Planck's Constant;
c is the speed of light;
G is the Gravitational Constant;
m_H is the mass of a Hydrogen Atom;

(All other constants not easily obtainable in the formula booklet are already provided in the equation above)

Question III

With this in mind, calculate the Chandrasekhar limit for the white dwarf, in terms of Solar masses. [2 marks]

Carrie, on the other hand, also spent 3 years analysing periods of countless stars within Cα-Kε, and identified many regular Cepheid variables near the centre of the galaxy. She worked aboard the International Space Station and had a much better view of Cα-Kε than Ken does. To adjust for issues with instrument sensitivity, she realised that **0.15 should be subtracted from the difference between apparent and absolute magnitude.**

Cepheid variable observed**	Luminosity (Apparent Magnitude)*			Period (days)	Location of Cepheid in Cα-Kε
	Maximum	Minimum	Average		
Cα – 1	23.8	26.2	25.0	53.1	Near top of Galactic Bulge
Cα – 2	25.1	26.5	25.8	43.2	Galaxy Arm
Cα – 3	25.9	27.1	26.5	30.4	Edge of Galactic disk
Cα – 4	25.4	26.4	25.9	26.2	Empty region between Arms

*Disclaimer 3: Remember to subtract 0.15 from the final value used for the *difference between apparent and absolute magnitude*.

**Disclaimer 4: These are fictional stars with data from actual stars

Question IV

(a) Help Carrie write a short message to Ken on what Cepheid variables are, and how they are used to calculate distances in space [2 marks].

(b) Using the above data, obtain a suitable estimation of the distance between Earth and Cα-Kε. Provide rationale for which data point(s) is/are used in obtaining your estimation. (Hint: Teamwork required) [4 marks]

(c) Comment on two OTHER possible sources of error (besides Ken's and her instrument) which could have affected Carrie's estimate of the distance between us and Cα-Kε, using Cepheid variables. [2 marks]

(d) Comment on any discrepancy between the results from Ken's and Carrie's methods, and suggest one other possible method to verify the actual distance. [2 marks]

Plot Twist!

If you've followed the story thus far, the Astrochallenge 2016 team will like you to vote for your preferred plot twist and ending. Please indicate it EITHER next to your School and Team name, OR at the end of the answers to this question. Please refrain from writing a new plot twist unless your team is confident of finishing the rest of this paper.

The plot twists are as follows:

A. Igor Nance turned out to be a mad scientist, and the villain.

B. Cα-Kε turns out to be M100 all along, but somehow everyone forgot about it.

C. Carrie turns out to be an artificial intelligence who has Ken fooled.

D. The world is but one of many multiverses which will fail to exist.

E. Carrie and Ken reunite and live happily ever after.

F. Others (Create your own ending)

DRQ 2: Tabby's Star [20 marks]

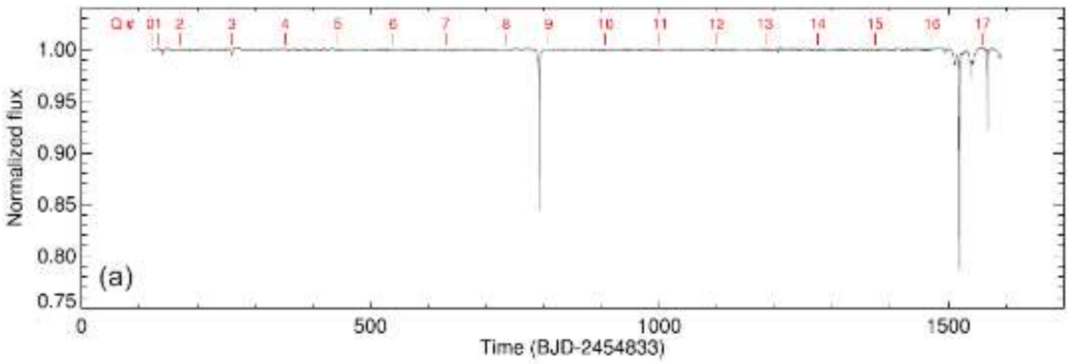
KIC 8462852, otherwise known as Tabby's star, has been widely publicized for its unusual and erratic behaviour. In this question, we will try to dispel some erroneous myths that have been going around. Before we begin, here are some known properties of Tabby's Star:

Mass	1.43 $M_\odot$
Radius	1.58 $R_\odot$
Bolometric Luminosity	4.7 $L_\odot$
Distance	454 pc
Apparent magnitude	11.7

While the age of the star is unknown, its spectral signature suggests that it is a main-sequence star. Furthermore, kinematic studies suggest that the star is not associated with any nearby newborn star clusters.

Kepler Light Curve

A portion of the Kepler light curve for Tabby's Star is shown below. As we can see, the key feature of Tabby's star is that it displays large dips in its light curve that can last for several days.



During the largest dip, the observed luminosity fell by 22%. To see how astounding that is, let us consider the following scenarios:

Question I

(a) If these dips are caused by a single body orbiting the star, its orbital period is around 750 days. What is the semi-major axis of the body? [1 mark]

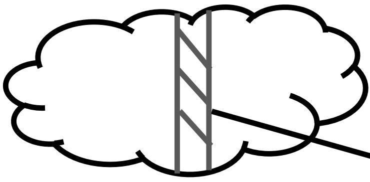
(b) Suppose a planet was responsible for the dip seen here. Determine its hypothetical radius, in terms of Jupiter radii. [2 marks]

(c) What would be the blackbody temperature of this "planet"? [3 marks]

(d) Given that it is relatively cool, the large apparent size of this "planet" is probably not due to the presence of an escaping atmosphere. Furthermore, planetary models suggest that the maximum size (by volume) of a planet is roughly the size of Jupiter. Why is this the case? In other words, why wouldn't adding a large amount of mass to Jupiter increase its volume significantly? Thus, explain why a planet is unlikely to cause this dip. [3 marks]

Question II

Instead of a planet, suppose a dust cloud was responsible for the observed dip. For simplicity, assume the dust cloud is similar to that of the Martian atmosphere. The column mass of the Martian atmosphere is 160 kg per square meter covered, and assume that the Martian atmosphere blocks off 40% of all incoming visible radiation.



A column with a $1m^2$ base passing through this gas cloud contains a total mass of 160 kg

Under these assumptions, what would be the mass of the dust cloud that lies in our line of sight? Express your answer in terms of Moon masses. [2 marks]

Photometry with Harvard Archival Plates

Deepening the mystery, it turns out we do have long term photometric observations of the star, along with 2 nearby field stars. As we can see below, Tabby's star (diamonds) displays a noticeable long-term dimming trend, especially compared to nearby stars.

Note: m_B is the apparent magnitude of a star, measured through a standard blue (B) filter

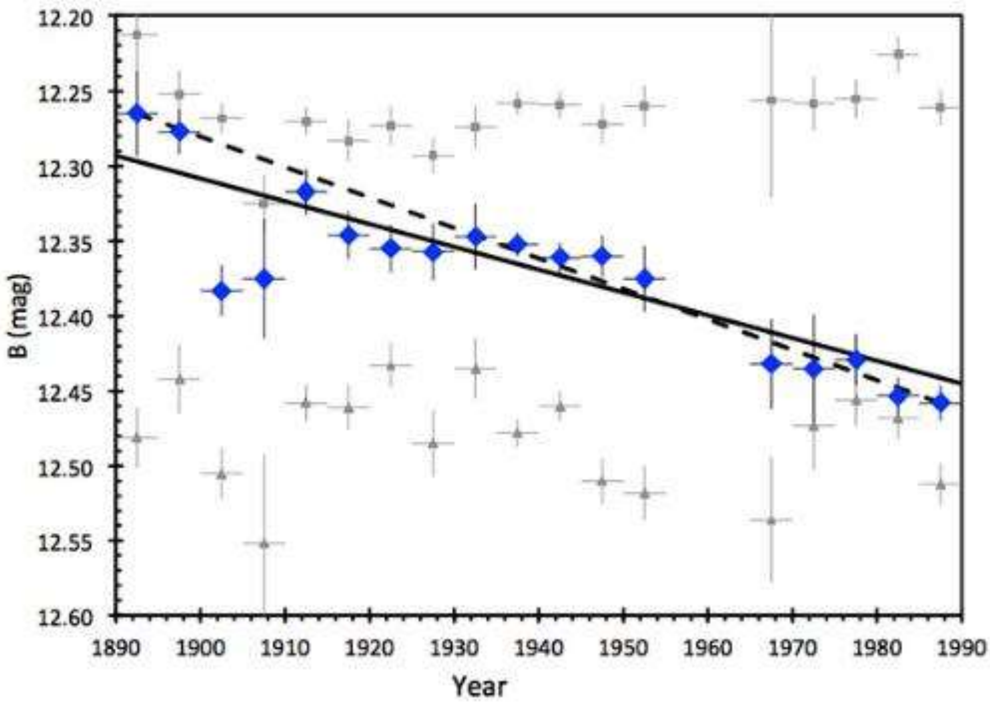


Figure 2.1 *Photometric graph showing the long-term dimming of Tabby's star*

Question III

(a) Between 1890-1895, Tabby's star had an average m_B magnitude of 12.265, which fell to 12.458 between 1985-1990. Assuming this trend is repeated for the star's actual luminosity, what is the average change in the star's luminosity per year over this 100-year interval? [4 marks]

(b) In the previous section, we assumed that the change in the star's m_B fully reflects changes in the star's luminosity. Is it possible for this assumption to be false? Explain using an example. [3 marks]

(c) The current leading explanation for the large short term dips observed in the star is that we are observing a swarm of comets evaporating very near the star. Briefly explain why this hypothesis has difficulty explaining the long term trends in this light curve. [2 marks]

DRQ 3: The Hertzsprung-Russell (H-R) Diagram [20 marks]

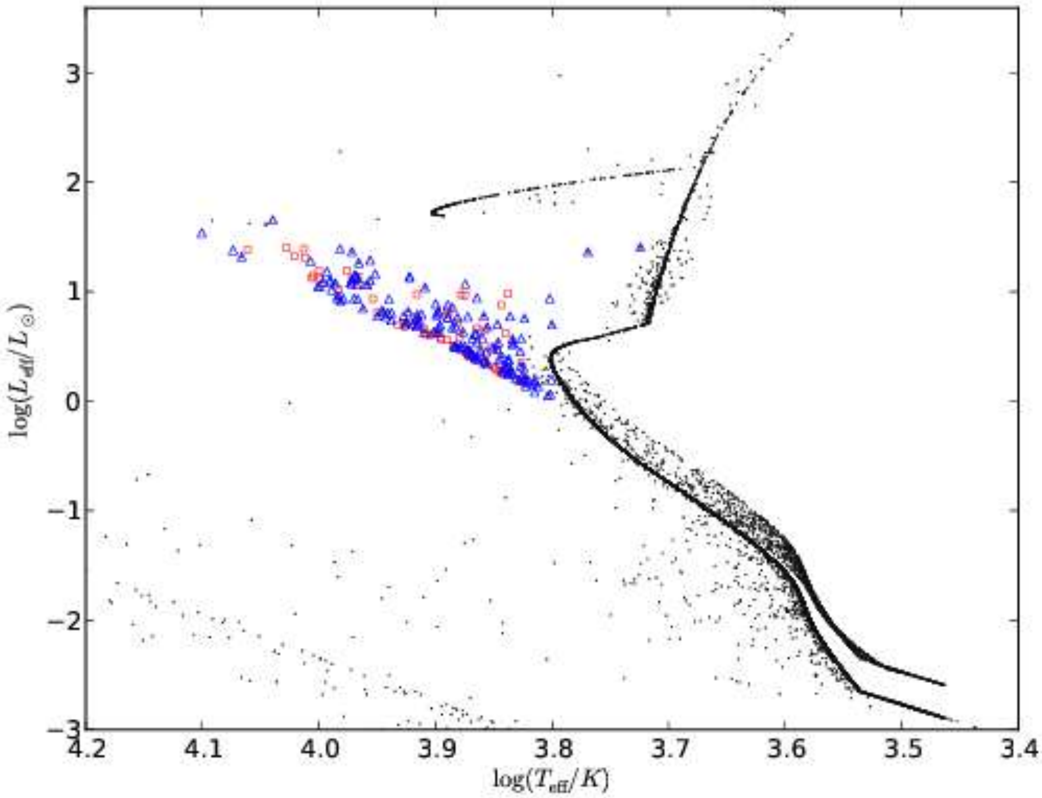


Figure 3.1 *shows the Hertzprung-Russell (H-R) diagram, plotting luminosity versus temperature, from a simulation. Squares and triangles denote blue-straggler stars in single and binary systems, respectively. Source: Chatterjee (2013)*

The existence of "blue stragglers" has confounded astronomers ever since their existence was first noted in Allan Sandage's 1953 paper. Although they are relatively rare stars, a few of them have been found in star clusters, where they clearly stand out because they occupy a region of the H-R diagram that should be completely empty. A star's age and initial mass uniquely determine its luminosity and temperature: stars of a given mass on the main sequence have a more or less fixed position on an H-R diagram. In a star cluster, all members formed at about the same time, and so we see a clear position on a cluster H-R diagram called the "turn-off" where stars with main sequence lifetimes equal to the age of the cluster starts to move off. This phenomenon occurs with the exception of a few interlopers affectionately dubbed as "blue stragglers".

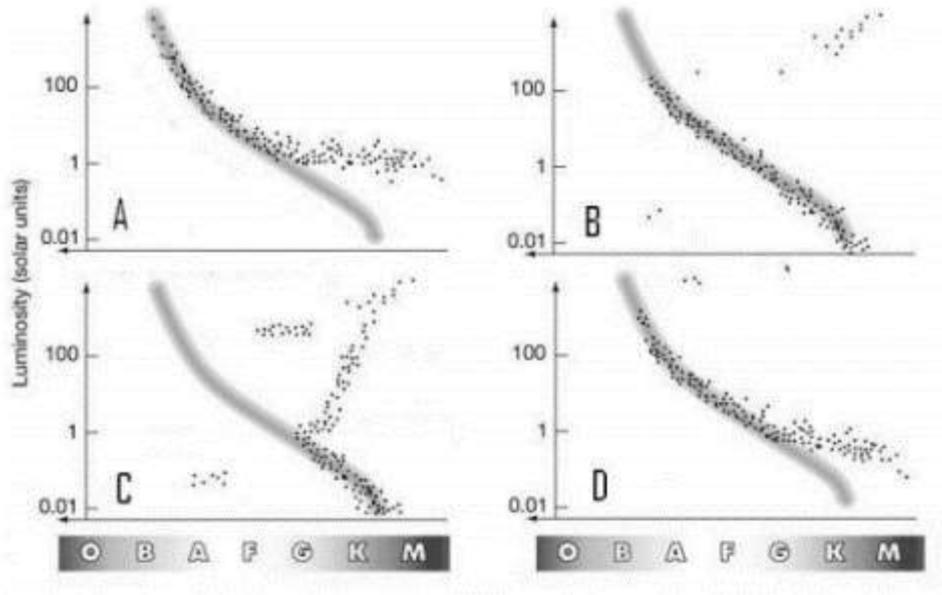


Figure 3.2 *4 H-R diagrams showing how stars of different masses (all starting from the same age) evolve over time. Each dot in the diagrams represents a star.*

<u>Question I</u>

(a) From Figure 3.1, determine the absolute bolometric magnitude of the main-sequence turnoff point. [2 marks]

(b) It is believed that the most luminous red giant stars serve as a standard candle. Define what a standard candle is. [1 mark]

(c) Using Figure 3.1, determine the luminosity and absolute bolometric magnitude of the brightest red giant. [2 marks]

(d) Rank the H-R diagrams in Figure 3.2 according to increasing age. Explain the motivation for your choice. [3 marks]

(e) Suggest why stars of spectral class M lie above the main-sequence in diagrams A and D [3 marks]

(f) Why is it surprising for astronomers to find “blue stragglers” beyond the ‘turn-off point’? [2 marks]

(g) Explain how blue stragglers are formed. [2 marks]

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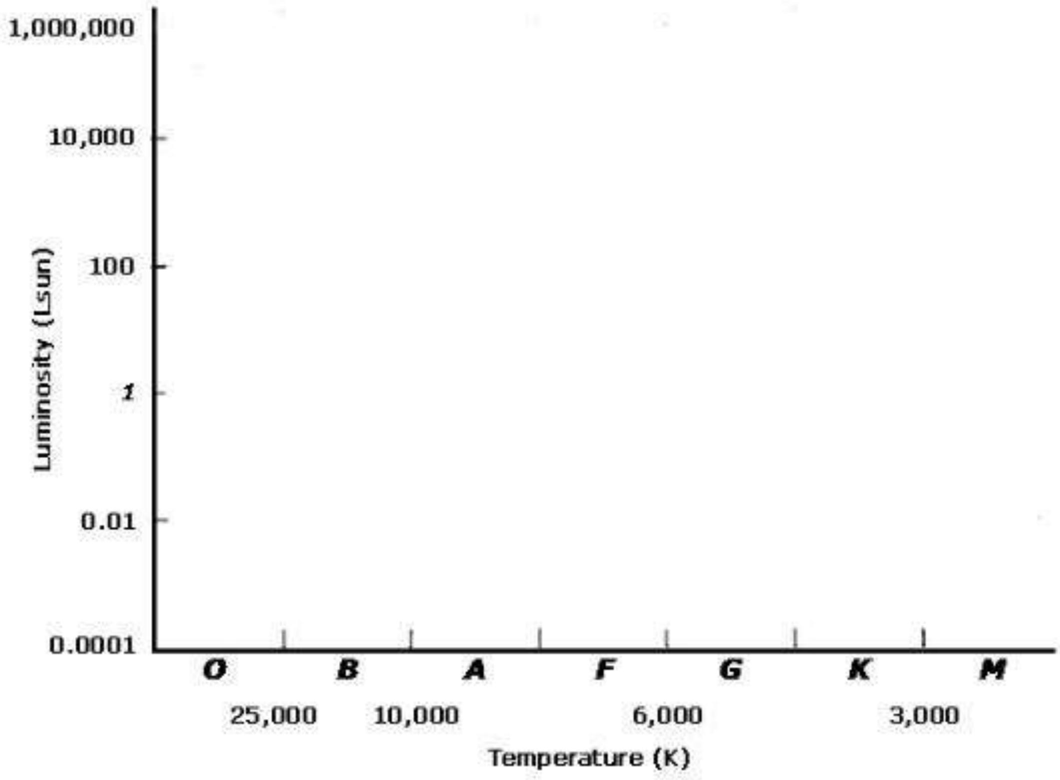

Figure 3.3 *H-R Diagram for Luminosity against Temperature*

Question II

(a) Find an expression for Log_{10} (L/L_{sun}) in terms of radius R/R_{sun} and T_{eff} and other constants. [2 marks]

(b) Sketch the equation found in (a) on Figure 3.3 for three different scenarios, namely R = R1, R2 and R3, where R1 < R2 < R3. Label your sketches.. [3 marks]

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DRQ 4: The Martian [20 marks]

In the movie "The Martian", *Hermes* took on itself to return to Mars to pick up astronaut Mark Watney who had been stuck on Mars for 47 Sols. *Hermes* did so using the so-called "Rich Purnell manoeuvre". Similarly, we are on a space mission from Earth to Mars that is not too far different from the said manoeuvre. Our mission is to launch a spacecraft from Earth to Mars in the period of 2016-2020. The Hohmann transfer orbit method is used for this mission. In this question, you will be investigating whether the mission will be successful.

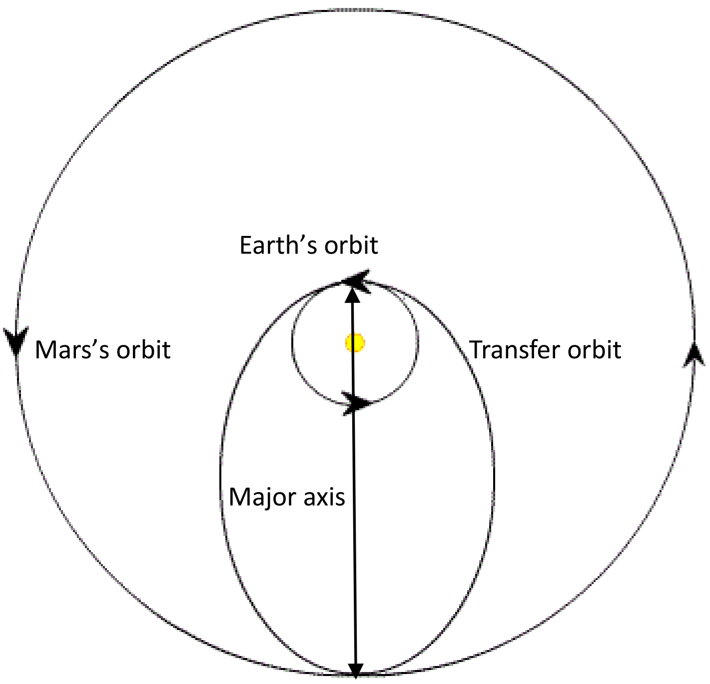


Figure 4.1 *This diagram shows Sun in the center of it all, the inner orbit represents in the orbit of Earth and the outer orbit that of Mars.*

The Hohmann transfer orbit is an **elliptical orbit** used to transfer between two circular orbits of different radii on the same plane. To perform the Hohmann transfer, 2 engine impulses (instantaneous velocity changes, Δv) are required. The first impulse moves the spacecraft onto the elliptical (intermediate) orbit and the second impulse moves it off into the orbit of the other planet/object.

In general, the total energy of a spacecraft is the sum of its ***kinetic energy*** and ***gravitational potential energy***, and this total energy also equals ***half the gravitational potential energy at the "average" distance*** a (the semi-major axis of the elliptical orbit).

Question I

(a) Write down the equation in relation to the statement above and hence or otherwise express v^2, where v is the velocity of the orbiting body, in terms of the radius of earth's orbit (R_E), the mass of the sun (M_{sun}) and the semi-major axis, a and any other relevant constants. [2 marks]

The equation found in (a) is always otherwise known as the vis-viva equation. This provides us with an expression that relates the total energy of the body on the elliptical orbit at the point of perigee (the point closest to the Sun) in relation to its kinetic energy

(b) In order to find the first instantaneous change in velocity Δv_1, we will need to find the semi-major axis, a. It is known that for an ellipse, the semi-major axis is half the distance of the major axis, or the distance between the perigee and the apogee as shown in the diagram below. Hence, express semi-major axis, a as a function of the earth's orbit, R_E, as well as Mar's orbit, R_M. [1 mark]

(c) Before launch, the initial velocity of the spacecraft is equal to the Earth's orbital velocity. From the information above, show that the first instantaneous change in velocity Δv_1 required to transfer into the Hohmann orbit is:

$$\Delta v_1 = \sqrt{\frac{GM_{Sun}}{R_E}} \left(\sqrt{\frac{2R_M}{R_E + R_M}} - 1 \right)$$

[4 marks]

For the spacecraft to leave the Hohmann transfer orbit onto Mars' orbit, a second instantaneous change in velocity Δv_2 is required. The idea is similar as found in the previous steps (a) – (c) except that we are moving OUT of the Hohmann transfer orbit. This yields:

$$\Delta v_2 = \sqrt{\frac{GM_{Sun}}{R_M}} \left(1 - \sqrt{\frac{2R_E}{R_E + R_M}} \right)$$

(d) To calculate fuel requirements (and associated initial mass of the spacecraft, m_i). I use two different methods.

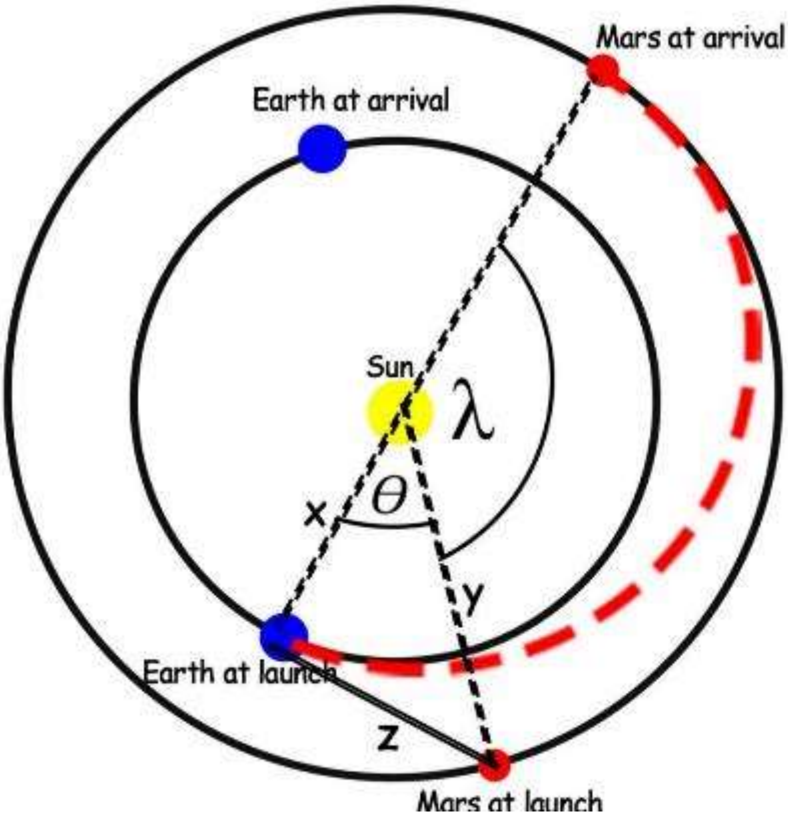
- Apply the rocket equation twice, using Δv_1 and Δv_2 as required, and changing the final mass of the spacecraft for each stage.
- Apply the rocket equation once, but using $\Delta v_T = \Delta v_1 + \Delta v_2$ instead.

Show that both methods will lead to the same value for m_i [2 marks]

(e) Hence or otherwise, given that $v_{\mathrm{exh}} = 2989.1$m/s, find the percentage loss in mass for the 2 impulses (instantaneous change in velocity), Δv_1 and Δv_2. [2 mark]

Some details of the initial plan are as follow:

- The spacecraft is to be launched on 8th June, 2016.
- On that day, the distance between Earth and Mars is 1.05×10^8km.



(f) Given the information provided, find angle θ and hence angle λ as shown in the diagram above. [2 marks]

Hint: Triangle XYZ is generally NOT a right-angled triangle

(g) Using Kepler's Third Law, find how many days it will take for the spacecraft to reach Mars (or reach the apogee), supposing that it was launched on 8th June 2016. State any assumptions used. [2 marks]

(h) Given that the period of Mars orbit is 687 Earth days, suggest if the spacecraft will reach Mars in time if it was launched on 8th June 2016. [3 marks]

DRQ 5: Observation Question [20 marks]

One fateful Friday evening, Ivan decided to do a sidewalk on the Engineering Bridge, Faculty of Engineering, National University of Singapore. He brought along a 5" Newtonian with a focal length of 700mm and sadly only an eyepiece with a focal length of 30 mm and a true field of view of 55 degrees.

Ivan only realised it was a full moon after setting up and hence grudgingly pointed at the full moon for viewing.

Question I

(a) What is the angular diameter of the Moon? Assume circular orbits. [2 marks]
(b) Determine the apparent field of view of the eyepiece, and hence determine if the Moon fills the frame of the eyepiece with this setup. [3 marks]
(c) During the sidewalk, a student asks what a zodiac constellation is. Explain. In other words, what property makes the zodiac constellations different from the rest?
[1 mark]

Despite the full moon, Ivan decided he would give a Stellarium show of "What-Could-Have-Been-Seen", if the moon won't there. He then orientated the view on Stellarium to this particular part of the night sky as shown on the next page.

Question II

(a) Identify the cardinal points on the diagram on the next page [2 marks]
 i) Circle Polaris (α UMi) [1 mark]
 ii) Trace out the 'Southern Cross' and label it accordingly. [1 mark]
 iii) Trace out the 'Summer Triangle' and label it accordingly. [1 mark]
 iv) Trace the 'Big Dipper' and label it accordingly [1 mark]
(b) List out 2 major IAU constellations (other than Crux, Ursa Major and Ursa Minor) that can be seen in the diagram. [2 marks]
(c) List down 2 prominent nebulae that are visible, and mark their positions on the diagram. [2 marks]
(d) List down 2 prominent open clusters that are visible, and mark their positions on the diagram. [2 marks]
(e) List down 2 prominent globular clusters that are visible, and mark their approximate positions on the diagram. [2 marks]

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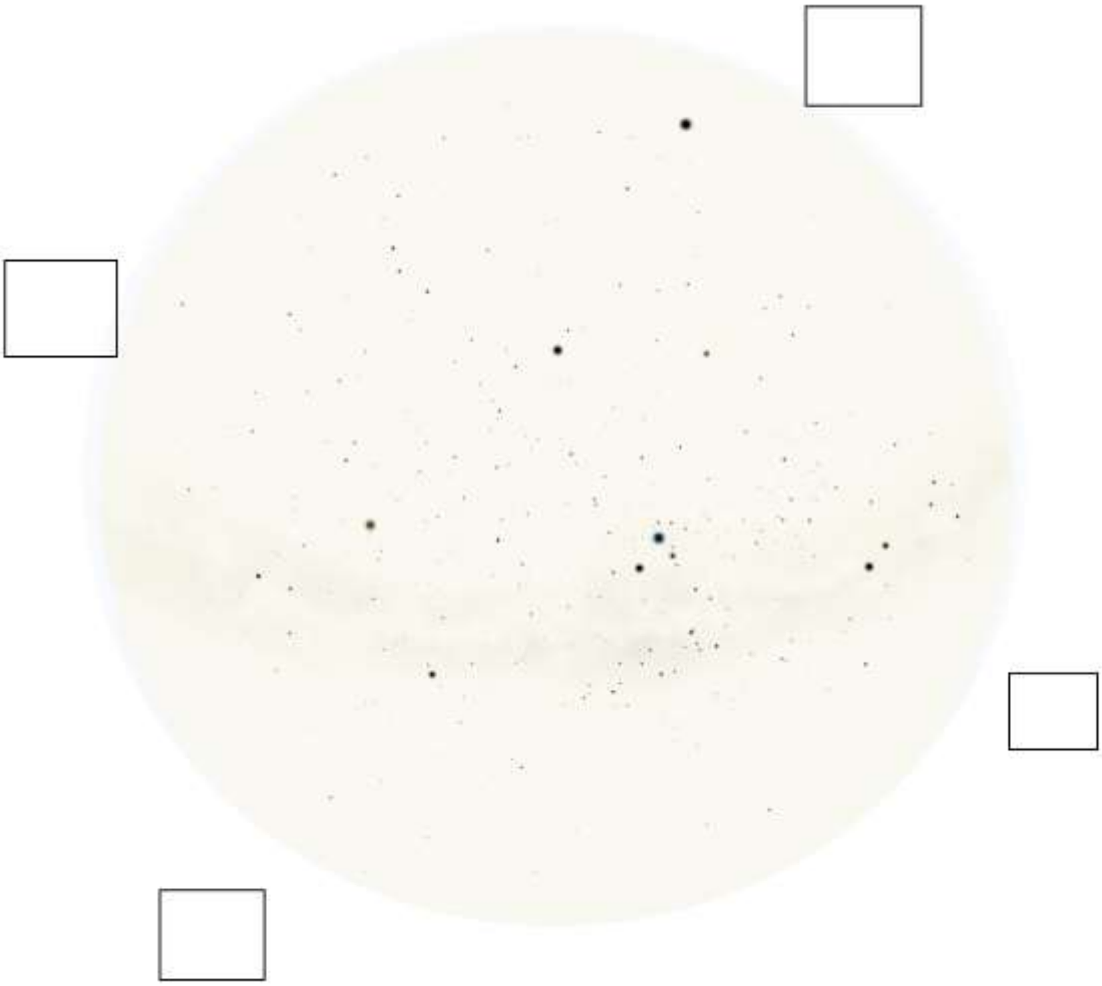

The figure shows approximately half of the celestial sphere, but which half?

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