



ASTROCHALLENGE 2016 DATA RESPONSE QUESTIONS

SENIOR ROUND

INSTRUCTIONS

- This paper consists of 12 printed pages, excluding this cover page and appendices.
- Do **NOT** turn over this page until instructed to do so.
- You have 2 hours to finish all questions in this paper.
- At the end of the paper, submit this booklet together with your answer script.
- Your answer script should clearly indicate your school (and team number) on **EVERY** page, as well as the individuals in the said team on the first page.
- It is your team's responsibility to ensure that all pages of your answer script have been submitted.

DRQ 1: Astronomical Reasoning [20 marks]

Question I

Rank these locations by increasing amount of twilight during the June solstice. Justify your answers. [4 marks]

- An observer at 45°N
- An observer at the equator
- An observer at 45°S

You are given that twilight occurs only if the sun is between 0° to 18° below the horizon.

Hint: a formal mathematical proof is overkill, simple sketches with explanations will suffice.

Question II

With suitable calculations, estimate the thickness of our Galactic Bulge based on the following diagram. State all of your assumptions **explicitly**. [3 marks]

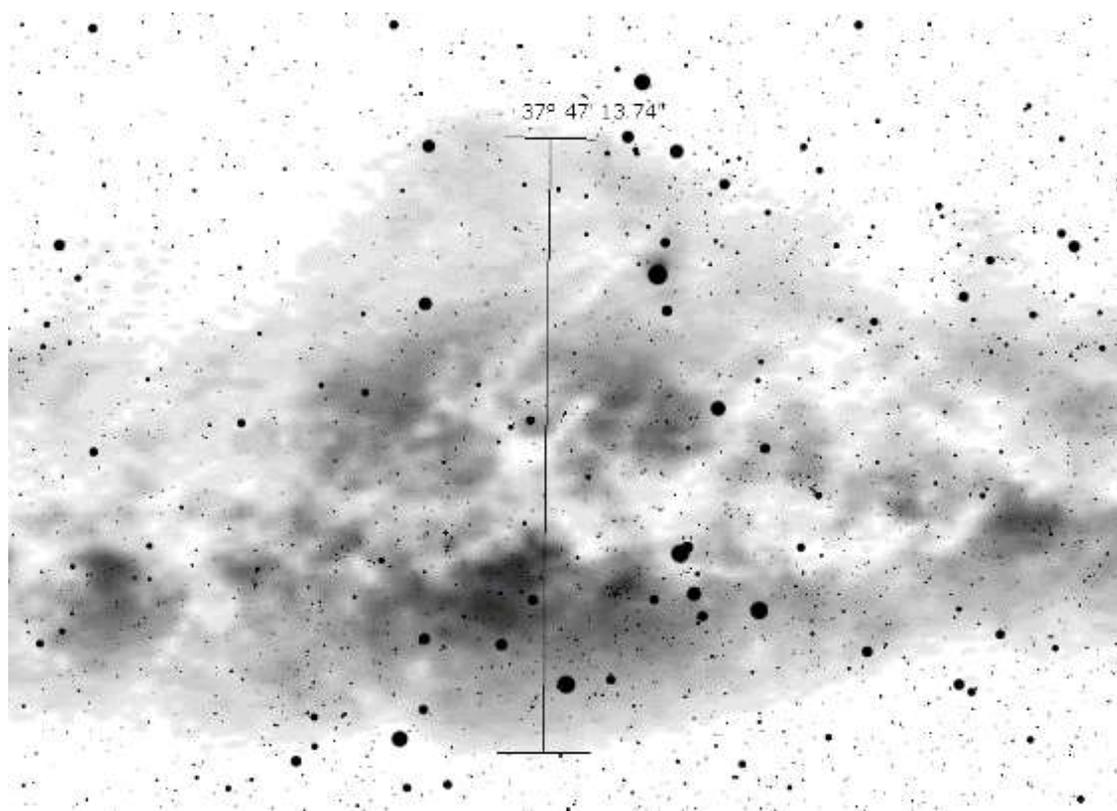


Figure 1.1 A simulated widefield view of the Milky Way core.
Its colours have been *inverted* for clarity.

Question III

- (a) I have a telescope with an aperture of X mm, set up such that it has a magnification of Y times. Given that a dark adapted pupil has a diameter of 7mm and has a limiting magnitude of 6, what is the limiting magnitude of a star through this setup? [3 marks]
- (b) I then turn to a planetary nebula with a surface brightness of S_{neb} and an area of A square arcseconds. The whole planetary nebula also has an apparent integrated magnitude m_{neb} . Express m_{neb} in terms of A and S_{neb} . [2 marks]
- (c) Through the scope, the apparent integrated magnitude of the planetary will increase by the same factor in part (a). Hence, find the new surface brightness S in terms of S_{neb} , X and Y . [3 marks]

Hint: If something is magnified by Y times, its apparent diameter increases by a factor of Y .

Question IV

Recall the Jean's Length formula:

$$R_J = \sqrt{\frac{15k_b T}{4\pi G \langle m \rangle \langle \rho \rangle}}$$

Where:

- k_b represents the Boltzmann constant
- T is the temperature of the gas cloud
- G represents the gravitational constant
- $\langle m \rangle$ represents the mass per particle (a constant)
- $\langle \rho \rangle$ represents the density of the cloud

Now, suppose we have a molecular cloud with a uniform density. It is suddenly forced to compress such that its diameter decreases by a factor of n . During this process, the temperature of the cloud remains constant.

- a) Find R_J' (the new value of R_J) in terms of R_J and n . [2 marks]
- b) Hence, explain why stars tend to form in clusters, even in the absence of other star forming mechanisms (e.g. supernovae shock waves and stellar winds). [3 marks]

DRQ 2: Mass-Luminosity Relation [20 marks]

The relationship between the mass of a star and its Luminosity is an important feature which allows use to calibrate our mass measurements. However, this relationship is an empirical relationship. In order to establish a theoretical basis for this relationship, we start off with a massive star filled with an ideal gas of the same temperature throughout.

Question I

- (a) Given that the Ideal-Gas Law is

$$PV = NkT$$

and that from integrating the hydrostatic equilibrium equation:

$$P \sim \frac{GM^2}{R^4}$$

find temperature T in terms of mass M , average mass \bar{m} , density, ρ . [2 marks]

*Note that number density,

$$n = \frac{N}{V} = \frac{\rho}{\bar{m}}$$

- (b) Hence or otherwise, express luminosity L in terms of only mass M and other constants. [3 marks]

We will now attempt to empirically affirm the results found previously. On **Appendix A**, attached at the back of this paper, you will find the data for some of the stars found in the Great Nebula in Orion, M42. In general, we believe that:

$$L \propto M^\alpha$$

where α depends on the mass of the star on the Main Sequence Branch. This implies that:

$$\frac{L}{L_{sun}} = \left(\frac{M}{M_{sun}}\right)^\alpha \rightarrow \log \frac{L}{L_{sun}} = \alpha \log \frac{M}{M_{sun}}$$

Question II

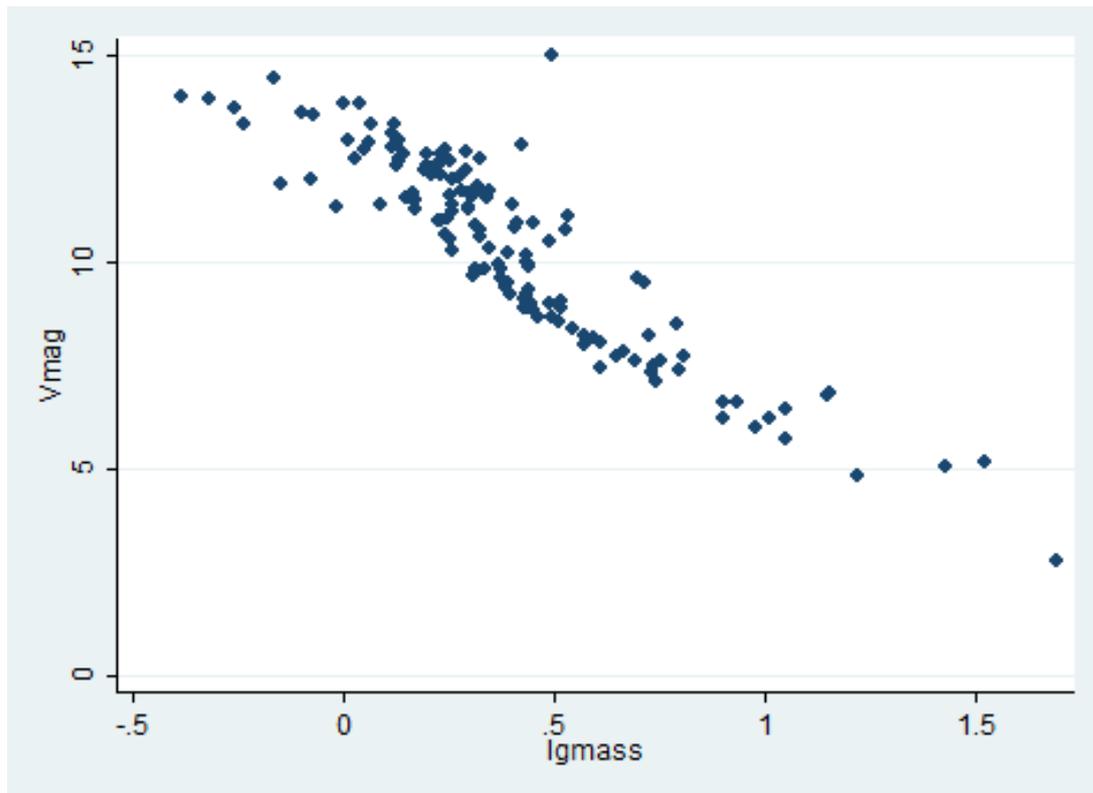
- (a) Given the data in Appendix A, plot a line of best fit on the graph paper provided and estimate the constant α . [8 marks]

Question III

If the general mass-luminosity relationship is true, then there should be a relationship between the apparent bolometric magnitude m , and mass M .

- (a) Express m as a function of M , α , distance d and other suitable constants. [2 marks]

Using the full dataset of stars in M42, a student has plotted the following graph:



Through ordinary least squares, the student claims that the following linear relationship is the best fit to the data:

$$m_v = 12.8 - 6.2 \log M$$

Where m_v represents the apparent visual magnitude of the star, as measured through a standard V filter in the UBV photometric system. Log M here refers to the mass of the star, in solar units.

Question IV

- (a) Using your answer in III(a), determine the implied value of α and d [2 marks]
- (b) You should find that your two estimates of α are inconsistent. Which estimate of α is likely to be more accurate? [1 mark]
- (c) Why is the other estimated value of α so wildly wrong? [2 marks]

DRQ 3: The Hertzsprung-Russell Diagram [20 marks]

When we plot a classical H-R diagram of a cluster, we see a clear “turn-off” where stars are moving into the red giant phase. However, in modern observational versions of the H-R diagram, the temperature (or spectral type) scale on the HR diagram is replaced by the colour index, and the luminosity is replaced by the apparent magnitude to produce a colour-magnitude diagram.

How do we measure “colour”? We do so by taking 2 images of the star through 2 standard filters, giving what is known a color index. Today, most often, the B-V colour index is employed, referring to the difference in apparent magnitudes through a B (blue) and V (visual) filter. Under this system, a blue star will have negative colour index (as its brighter through the B filter than the V filter); while a yellow or red star will have positive colour index. Hence, the color index gives us a gauge of the star’s temperature.

For the purpose of our discussion, we will be focusing on the $g'-r'$ index for the SLOAN narrowband filters which allows (peak) wavelength **477 nm** (green) and **6250nm** (red), with bandwidth of **140nm** individually (Fukugita, et al., 1996).

Question I

- (a) Other than color, luminosity and temperature, suggest another piece of information a color-magnitude diagram can yield. [1 mark]
- (b) Explain how can we obtain this piece of information. [2 marks]

Question II

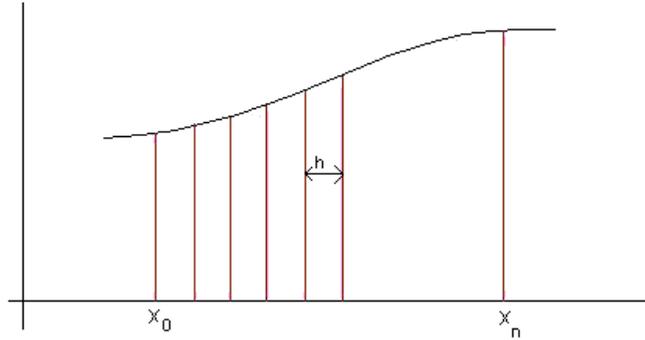
We will now attempt to prepare the tools required for the calculation of the $g' - r'$ value.

- (a) Given, $c = f\lambda$, find $\frac{d\lambda}{df}$. Then, prove that the following relationship is true:

$$I = \int_a^b \frac{2\pi hc^2}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1 \right)} d\lambda = \int_c^d \frac{2\pi h}{c^2} \left(\frac{f^3}{e^{\frac{hf}{kT}} - 1} \right) df$$

where I is the total intensity (aka luminosity per unit surface area) integrated over the entire emission spectrum of the star. [5 marks]

- (b) Knowing the bandwidth and the ‘peak wavelength’ observed through the filter allows us to calculate the observed intensities. Suggest appropriate integration limits to the definite integral above and find the supposed intensities of a star of temperature T for the two different narrowband filters, using the Trapezium Rule or otherwise. (continued on the next page)



Split the area under a curve into a number of trapeziums with some fixed area.

If we want to find the area under a curve between the points x_0 and x_n , we divide this interval up into smaller intervals, each of which has length h (see diagram above).

$$\int_{x_0}^{x_n} f(x) dx \approx \frac{1}{2}h((y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}))$$

where $y_0 = f(x_0)$ and $y_1 = f(x_1)$ etc.

- (i) Suggest an appropriate value for n and hence width h for the integration. [1 mark]
- (ii) Suggest appropriate integration limits (a and b) for the equation found in II(a). [1 mark]
- (iii) State the assumption required for your answer in part (ii). [1 mark]
- (iv) Hence, calculate the respective intensities for the g' and r' filters ($I_{g'}$ and $I_{r'}$), given that $T = 6000\text{K}$ by summing the trapeziums. [7 marks]

Alternatively, you are more than welcome to do the integration directly to find the exact values. [10 marks]

Note: this is only for the **brave souls** with **too much time and brain power** to spare.

- It might be helpful first to note that $e^{\frac{hc}{\lambda kT}} \gg 1$ for $\lambda \ll 1$
- After the approximation, given that

$$\int \frac{hc}{kT\lambda^2} e^{-\frac{hc}{kT\lambda}} d\lambda = e^{-\frac{hc}{kT\lambda}}$$

- apply integration by parts 4 times

$$\int u dv = uv - \int v du$$

An increase in (apparent/absolute) magnitude corresponds to a decrease in brightness by a factor of 2.5. As previously mentioned, the magnitudes can be used to compare the apparent brightness of the two stars or against a reference. For two stars of magnitudes m_1 and m_2 and apparent brightness b_1 and b_2 , respectively, we have $\frac{b_1}{b_2} = 2.5^{m_2 - m_1}$. Applying this concept for one star instead of 2 but for 2 different intensity spectrum we may find $g' - r'$ expressed as the difference between the intensities.

- (i) State the equation of $g' - r'$ expressed in terms of the intensities. [1 mark]
- (ii) Hence calculate the $g' - r'$ value for the given star at 6000K . [1 mark]

DRQ 4: The Dark Side of Jupiter [20 marks]

After watching The Transformers: Dark Side of the Moon, Jyh Harnng asked during an observation session if it's possible to see the dark side of Jupiter as he was bored of looking at the bands of Jupiter and the Galilean moons through the telescope.

You will now investigate if it is indeed possible to see the dark side of Jupiter from Earth at any time of Jupiter's revolution about the Sun.

Question I

- (a) Sketch a diagram showing the Earth, Jupiter and Sun on the same plane, indicating the angle subtended between the Sun and Earth relative to Jupiter, as θ . [3 marks]
- (b) When will the angle θ be largest? [1 mark]
- (c) Calculate this largest value of angle θ . You may assume that the orbits of Earth and Jupiter are circular. [2 mark]
- (d) Hence or otherwise, calculate the illuminated phase of Jupiter in percentage of the surface area visible from Earth using the information found in (b) and (c). [2 marks]

Hint: Instead of calculating the illuminated surface area of Jupiter, work with longitudes. Under this system, if Jupiter is fully illuminated, we can see all 180° of its visible surface. The current illuminated phase of Jupiter is then a simple ratio, with the denominator being 180° .

Jyh Harnng was also eager to look out for the Great Red Spot (GRS) and wondered when the next time he might be able to see it on the surface of Jupiter would be so that he could image Jupiter with the GRS on it.

The table shows transit times of the Great Red Spot on Jupiter, for a certain year:

Date	Time
7 th June	04:23:00
7 th June	14:19:00
8 th June	00:15:00
8 th June	10:10:00
8 th June	20:06:00
9 th June	06:02:00
9 th June	15:58:00
10 th June	01:53:00
10 th June	11:49:00

Question II

- (a) Using the information provided above calculate the average period of rotation, T of the Great Red Spot. [2 marks]

- (b) Jupiter currently has an azimuth of 267° and an altitude of 24° . Hence, when would be the next time Jyh Harnng can see the Great Red Spot just appearing east of Jupiter? [4 marks]

Being a physics major, Jyh Harnng is now interested to calculate the tangential velocity of the Great Red Spot given the information above.

- (c) It is known that the Great Red Spot is 22° south of the equator on Jupiter, find the approximate tangential velocity, $v_{||}$ of the Great Red Spot. [1 mark]

Joel, a friend of Jyh Harnng who dreams of being an astronaut wonders how the Dark Side of Jupiter looks like. Joel, a physics major as well, decides to calculate the Hohmann transfer orbit required to arrive at Jupiter from Earth by neglecting the gravitational potential energy of Jupiter and Earth.

The Hohmann transfer orbit is an elliptical orbit used to transfer between two circular orbits of different radii on the same plane. To perform the Hohmann transfer, 2 engine impulses (instantaneous velocity changes, Δv) are required. The first one to move the spacecraft onto the transfer (intermediate) orbit and the second one to move it off into the orbit of the other planet/object.

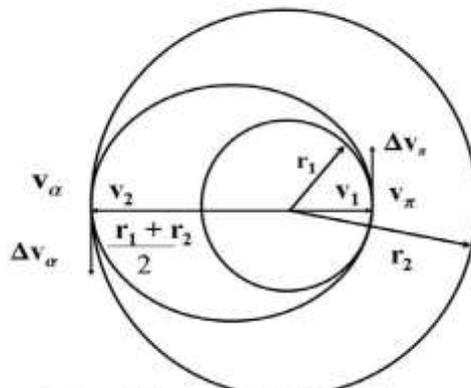


Figure 4.1 The Hohmann transfer orbit

Question III

- (a) It is known that the velocity of an object in circular orbit at some distance r around the sun is $\sqrt{\frac{GM_{\text{sun}}}{r}}$, while the velocity required to move into the elliptical orbit at $r = r_1$ from r_1 circular orbit is given as $v^2 = GM_{\text{sun}}\left(\frac{2}{r_1} - \frac{2}{r_1+r_2}\right)$. Given this information, calculate Δv_1 for the **first impulse** and Δv_2 for the **second impulse**. [3 marks]
- (b) Given that $v_{\text{exh}} = 2989.1$ m/s find the total change in mass for the 2 impulses. [2 marks]

DRQ 5: Lost in a Forest Far Far Away [20 + 2 bonus marks]

Not long ago, there was a group of young astronomers camping in a forest. While they were setting up a telescope, dark clouds began to roll in. Worried, they ran for shelter, and found a road leading to an observatory on top of a faraway hill. As thunder roared in the distance, they found an entrance into the observatory. The hallways were dimly lit without a single person in sight. As the group progressed, they found a monument scribbled with the following formula:

$$\frac{1}{\lambda_{vac}} = R_H \left(\frac{1}{N_1^2} - \frac{1}{N_2^2} \right)$$

Equation 1: Rydberg formula for Hydrogen

Where λ_{vac} is the wavelength of electromagnetic radiation emitted in vacuum (in meters, m),
 R_H is the Rydberg constant for Hydrogen, approximately $1.097 \times 10^7 \text{ m}^{-1}$,
 N_1 is the principal quantum number of the orbitals occupied after
 N_2 is the principal quantum number of the orbitals occupied before

As lightning flashed across the sky, the team decided to enter a door labelled 'Lyman', hoping to find someone. Alas, all they could find was another equation scribbled on the desk with a note: "Should you have absolutely no idea what is going on, read up the appendix for a crash course on the Hydrogen atom. "

Could they solve this mystery?

Question 1

(a) Show that Lyman series where $N_1 = 1$, the Rydberg equation can be expressed as:

$$\lambda_{vac} = \left[R_H \left(1 - \frac{1}{N_2^2} \right) \right]^{-1}$$

[0.5 mark]

(b) Calculate the peak emission wavelength (in nm) of the Lyman alpha spectral line ($\text{Ly}\alpha$), where $N_2 = 2$. [0.5 mark]

As the team looked out of the window, they are greeted by the sight of a dim, misty and rainy forest. On the table next to the equation is an extremely messy graph. One student with some knowledge about cosmology recognised it as a spectrum of an extremely distant quasar. There's no mistake – the telescope and spectroscopy equipment were measuring this high redshift quasar. The $\text{Ly}\alpha$ line, the most prominent emission line of Hydrogen, is measured, but interestingly it is followed by many $\text{Ly}\alpha$ absorption lines, as shown in the next page.

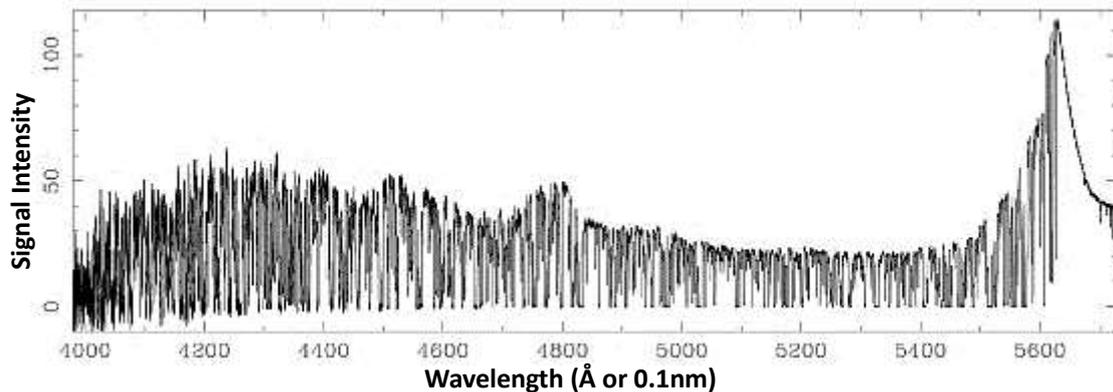


Figure 5.1 Lyman alpha forest characteristic of the spectrum of a distant, high redshift quasar. The peak Ly α emission wavelength is at 563 nm, followed by the messy ‘forest’ of shorter Ly α absorption wavelengths. Adapted from Rauch, 1998.

Question II

- Briefly explain how Cosmological Redshift occurs (Hint: this phenomena resulted in the difference between your Ly α value from (a) (ii) and the above graph). [1.5 marks]
- Calculate the redshift of the quasar [0.5 mark].
- Show that the velocity of the quasar as a fraction of the speed of light (v/c) based on relativistic redshift (formula in formula booklet) can be expressed in the following form:

$$\frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

[3 marks]

- Using the redshift from (b) and the equation from (c), calculate the velocity of the quasar in terms of c (speed of light), and write down any comments/ interesting observations about this value. [1 mark]
- Calculate the distance by which the quasar is away from us, taking H_0 to be 67.8km/s/Mpc. [1 mark]
- Assuming only empty space between Earth and this quasar, would you expect to see a Lyman Alpha forest? With this in mind, suggest what causes the Lyman alpha forest, as well as discuss the significance of this phenomenon. You may illustrate these concepts with the aid of another graph or diagram. (Hint: Think about the misty forest on a cosmological scale, and read the appendix if you’re lost) [4 marks]

The team then entered a door named ‘Balmer’. Oddly, when they looked out of the window, all they could see was a few distant mountains. Likewise, a note lies on the table, this time with a slightly different equation, and a large optical telescope is in the middle of the room next to a spectrometer.

For the Balmer series, where $N_1 = 2$, the Rydberg equation can be expressed as:

$$\lambda_{vac} = [R_H(0.25 - \frac{1}{N_2^2})]^{-1}$$

Question III

- Calculate the peak emission wavelength of the first four Balmer spectral lines, then show that all of them exist within the range of the visible spectrum (about 400 – 700 nm). [1 mark]
- In addition, a ultraviolet spectrometer report a spectral line reading of 301nm. The object is an old hydrogen lamp at rest. Could that line have been from the Balmer series? (Hint: proof by contradiction by first finding the Balmer limit: The minimum theoretical wavelength of the Balmer series.) [1.5 marks]

Similarly, a graph of two quasars is found on the table, but there are no supporting data on their identity. Nevertheless, since the peaks correspond to the Balmer series, they are probably not very far away compared to the high red shift quasar. Can they solve this mystery as well?

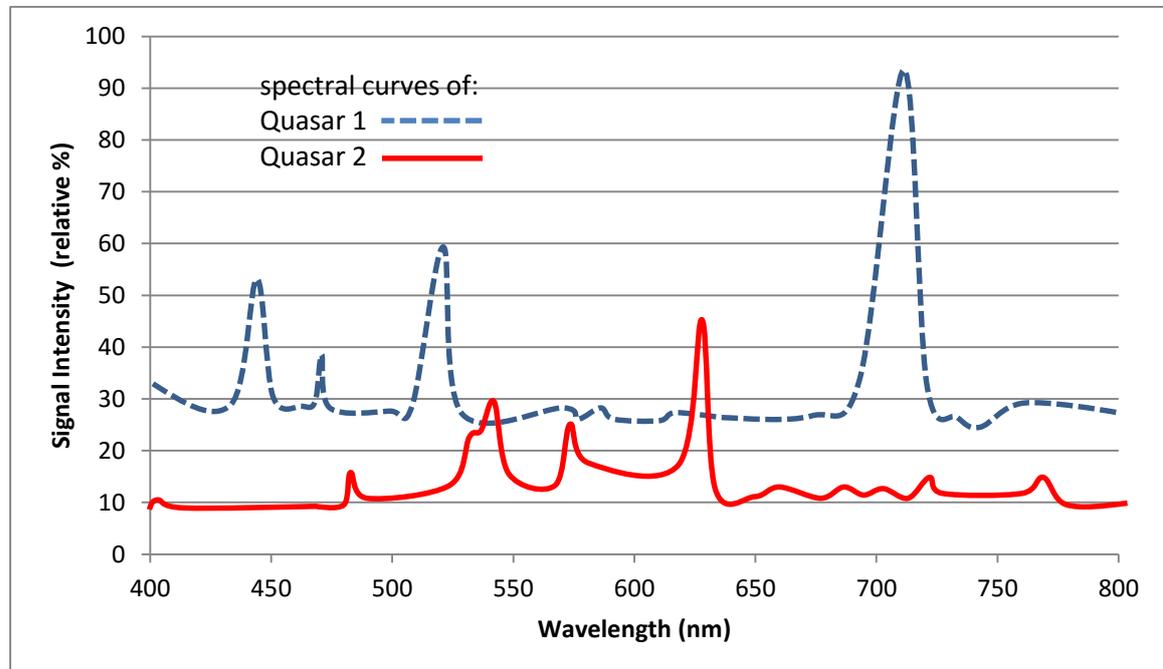


Figure 5.2 Balmer emission spectra for two hypothetical, relatively lower redshift quasars.

Question IV

- Calculate the redshift of both quasars using the Balmer series by estimating the values of the peaks from the graph, and hence determine which quasar is receding away from us at a faster velocity. [3 marks]
- Briefly explain the significance of using the Balmer Series over the Lyman Alpha series for low redshift quasars. [0.5 mark]
- Why are only three peaks present in Quasar 2? [0.5 mark]
- On their way back, the team passed by data pertaining to a third object, this time a star. This star is moving away from Earth at 200km/s. However, it is only about 100 Parsecs away from us. Prove that its movement is not a result of the expansion of the universe, and briefly suggest an alternative explanation for its high speed. [1.5 marks]

As the team gathered back in the lobby, they realised they have been on opposite ends of the observatory. In retrospect, they saw a misty forest in the Lyman room, but saw only mountains in the Balmer room. It struck them how coincidental it was that the graphs of each respective spectrum in the rooms ended up in a similar fashion. This leads to a final mystery, as heavy rain droplets begin to pour...

Bonus Question

Why would you observe a Lyman alpha forest, but not a Balmer alpha forest? [2 marks]

If you're reading this after (hopefully) surviving this ordeal, the Astrochallenge 2016 team will like you to vote for what you thought is the most frightening thing about this question. Please indicate it EITHER next to your School (and team number), OR at the end of the answers to this question. Do not spend too much time describing this section in graphic detail unless your team is confident of finishing the rest of this paper.

The options are:

- A. A lot more theory/ explanation questions than expected.
- B. Bad weather.
- C. Calculation questions were still tough.
- D. Difficulty in understanding content.
- E. Something else entirely – do indicate briefly.