DRCw 3

Hertzsprung - Russell Diagram

Q2. (a) $c = f\lambda \Rightarrow \lambda = \frac{c}{f}$

$$\frac{d\lambda}{df} = -\frac{c}{f^2}, \quad f = \frac{c}{\lambda}$$

$$df = -\frac{f^2}{c}\, d\lambda$$

$$J = \cancel{\int_a^b 2\pi h} \int_c^d \frac{2\pi h}{c^2}\left(\frac{f^3}{e^{\frac{hf}{kT}}-1}\right) df$$

$\Rightarrow$

$$= \int_{f=c}^{d} \frac{2\pi h}{c^2}\cancel{\left(\frac{c}{\lambda}\right)^3}\left(\frac{f^3}{e^{\frac{hc}{\lambda kT}}-1}\right)\cancel{df} \times \frac{d\lambda}{\cancel{df}}\left(-\frac{f^2}{c}\right)$$

$$= -\int_d^{f=c} \frac{2\pi h}{c^3}\frac{f^5}{e^{\frac{hc}{\lambda kT}}-1}\, d\lambda = \int_{\lambda=a}^{b} \frac{2\pi h}{c^3}\left(\frac{c}{\lambda}\right)^5\left(\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\right) d\lambda$$

$$= \int_{\lambda=a}^{b} \frac{2\pi h c^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\right) d\lambda \quad \text{(shown)}$$

$\Rightarrow$

(b) (i) $n \cancel{\geq} 20$

Accept $n \geq 3$ and appropriate value of h for the range.

(ii) g': { 547 nm (b), 407 nm (a) } r': { 695 nm (b), 559 nm (a) }

(iii) Assume that ~~he mean~~ the bandwidth is distributed evenly about the peak wavelength such that ~~8~~ 477 ± 70 nm and 655 ± 70 nm gives the correct limits.

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(iv) Marks awarded according to the number of ~~suppose your~~ η and open workings. } most not for the final value so much.

(b) Alternative:

$$I = \int_a^b \frac{2\pi hc^2}{\lambda^5 (e^{\frac{hc}{\lambda kT}} - 1)} \, d\lambda \qquad e^{\frac{hc}{\lambda kT}} >> 1 \text{ for } \lambda << 1,$$

$$I = \int_a^b \frac{hc}{\lambda^5} e^{-\frac{hc}{\lambda kT}} \, d\lambda \, (2\pi c) \qquad = 2\pi c \int_a^b kT \left(\frac{hc}{kT\lambda^2}\right) e^{-\frac{hc}{\lambda kT}} \frac{1}{\lambda^3}$$

$$= 2\pi ckT \int_a^b \frac{hc}{kT\lambda^2} e^{-\frac{hc}{\lambda kT}} \frac{1}{\lambda^3}$$

let $\frac{1}{\lambda^3}$ be u, $\frac{hc}{kT\lambda^2} e^{-\frac{hc}{\lambda kT}}$ be dv

$$= 2\pi ckT \left[\frac{1}{\lambda^3} e^{-\frac{hc}{\lambda kT}} \Big|_a^b + \int_a^b e^{-\frac{hc}{\lambda kT}} \frac{3}{\lambda^4} \, d\lambda \right]$$

continued ...

$$I = 2\pi ckT \left[\frac{1}{\lambda^3} e^{-\frac{hc}{\lambda kT}} \Big|_a^b + \frac{kT}{hc} \int_a^b \frac{hc}{\lambda^2 kT} e^{-\frac{hc}{\lambda kT}} \frac{3}{\lambda^2} \, d\lambda \right]$$

$$= 2\pi ckT \left[\frac{1}{\lambda^3} e^{-\frac{hc}{\lambda kT}} \Big|_a^b + \frac{kT}{hc} \left(\frac{3}{\lambda^2} e^{-\frac{hc}{\lambda kT}} \Big|_a^b + \int_a^b \frac{6}{\lambda^3} e^{-\frac{hc}{\lambda kT}} \, d\lambda \right) \right]$$

$$= 2\pi ckT \left[\frac{1}{\lambda^3} e^{-\frac{hc}{\lambda kT}} \Big|_a^b + \frac{kT}{hc} \left(\frac{3}{\lambda^2} e^{-\frac{hc}{\lambda kT}} \right)_a^b + \frac{kT}{hc} \int_a^b \frac{6}{\lambda^3} e^{-\frac{hc}{\lambda kT}} \, d\lambda \right]$$

$$= 2\pi ckT \left[\frac{1}{\lambda^3} e^{-\frac{hc}{\lambda kT}} \Big|_a^b + \frac{kT}{hc} \left(\frac{3}{\lambda^2} e^{-\frac{hc}{\lambda kT}} \right)_a^b + \left(\frac{kT}{hc}\right)^2 \frac{6}{\lambda} e^{-\frac{hc}{\lambda kT}} \Big|_a^b + \int_a^b \left(\frac{kT}{hc}\right)^2 \frac{6}{\lambda^2} e^{-\frac{hc}{\lambda kT}} \, d\lambda \right]$$

$$= 2\pi ckT \left[\frac{1}{\lambda^3} e^{-\frac{hc}{\lambda kT}} \Big|_a^b + \frac{kT}{hc} \left(\frac{3}{\lambda^2} e^{-\frac{hc}{\lambda kT}} \right)_a^b + \left(\frac{kT}{hc}\right)^2 \left(\frac{6}{\lambda} e^{-\frac{hc}{\lambda kT}} \right)_a^b + \left(\frac{kT}{hc}\right)^3 6 e^{-\frac{hc}{\lambda kT}} \Big|_a^b \right]$$

$$= 2\pi ckT \left[\frac{1}{\lambda^3} e^{-\frac{hc}{\lambda kT}} + \frac{kT}{hc} \left(\frac{3}{\lambda^2} e^{-\frac{hc}{\lambda kT}} \right) + \left(\frac{kT}{hc}\right)^2 \left(\frac{6}{\lambda} e^{-\frac{hc}{\lambda kT}} \right) + \left(\frac{kT}{hc}\right)^3 6 e^{-\frac{hc}{\lambda kT}} \right]_{\lambda = a}^{b}$$

$$= \left[\frac{12 (kT)^4 \pi}{c^2 h^3} e^{-\frac{hc}{\lambda kT}} + \frac{12 k^3 T^3 \pi}{\lambda c h^2} + \frac{6 k^2 T^2 \pi}{\lambda^2 h} e^{-\frac{hc}{\lambda kT}} + \frac{2\pi ckT}{\lambda^3} e^{-\frac{hc}{\lambda kT}} \right]_a^b$$

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for the green filter g',

$I_{g'} = 1.36576 \times 10^7$ after evaluating the integral
for $a = 407 nm$,
$b = 547 nm$

$I_{r'} = 1.16217 \times 10^7$ for $a = 557 nm$
$b = 695 nm$

)

(i) $\left(\frac{I_{g'}}{I_{r'}}\right)^{-1} = 2.5^{g'-r'}$

$\frac{I_{r'}}{I_{g'}} = 2.5^{g'-r'}$

$\Rightarrow 2.5 \lg\left(\frac{I_{r'}}{I_{g'}}\right) = g' - r'$

(ii) $g' - r' = -0.403568$

)

I

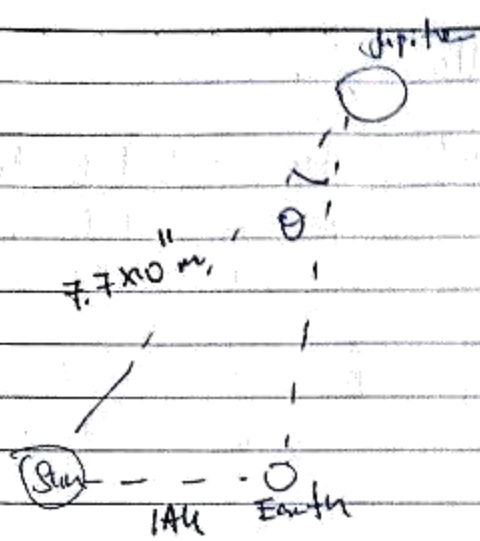
(a) distances / apparent magnitudes / spectral class / age of cluster

(b) distances from distance modulus (luminosity - apparent magnitude)
spectral class (fitting curve)
~~turn off point (age of cluster)~~

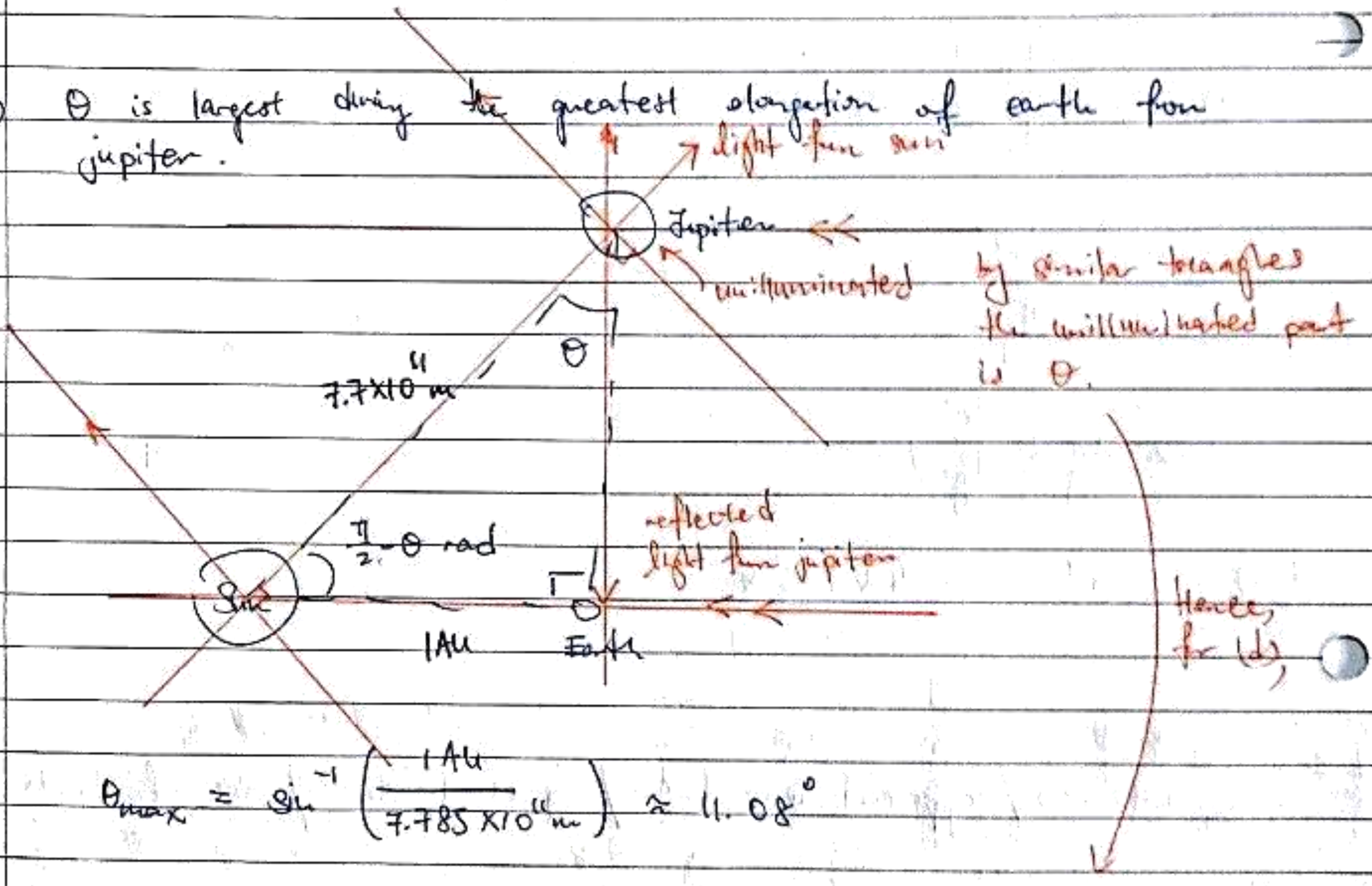
DRQ 4

I

(a)



(b) θ is largest during the greatest elongation of earth from jupiter.



(c) $\theta_{max} = \sin^{-1}\left(\frac{1AU}{7.785 \times 10^{11} m}\right) \approx 11.08^{\circ}$

(d) $\%\ \text{illumination} = \frac{180 - \theta_{max}}{180} \times 100\% = \cancel{0.938}\ 93.84\%$

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(II)

(a) 9h 56 minutes = Period.

(b) Given the info (Azi : 267°, alt : 24°)
Jupiter is going to set tonight (8th June)

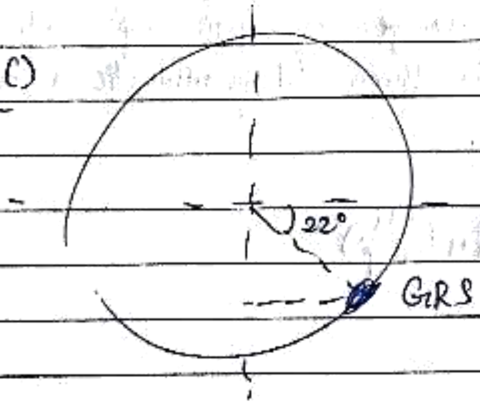
[an extra info was supposed to be given on the position of the GRS on the surface, however without which it is actually not possible to accurately solve the question.]

Can removed.

at the central meridian

Suppose it was given that GRS was ~~60° past the central meridian~~ on Jupiter we could solve further..
on 8th June that year, transit time is at 20:06 (night)
we would then find that 5/4 of period (rising in the east)
it would be 03:53. But, it would have set.
We could then possibly suggest an alternative time such as
on 9th June at 15:58 + 07:27 (3/4 of period)
to give 23:25 (again it is likely to have set too)
So, the best answer would be on 10th June, at
11:47 + 07:27 (3/4 of period) = 19:14.

(c)



$v_{\parallel} = R_{\parallel}\omega$, $\omega = \frac{2\pi}{T}$,

$T = 9h\ 56\ min$

$= 35760$ seconds

$v_{\parallel} = R\cos 22° \left(\frac{2\pi}{T}\right)$

$= 1.16 \times 10^{4}\ ms^{-1}$

Date No

III

(a) $\Delta v_1 = \sqrt{GM}\sqrt{\frac{2}{r_1}-\frac{2}{r_1+r_2}} - \sqrt{GM}\sqrt{\frac{1}{r_1}}$

$$= \sqrt{GM}\left(\sqrt{\frac{2}{r_1}-\frac{2}{r_1+r_2}} - \sqrt{\frac{1}{r_1}}\right) = \sqrt{GM}\left(\sqrt{\frac{2r_2}{r_1(r_1+r_2)}} - \sqrt{\frac{1}{r_1}}\right)$$

$$= \sqrt{\frac{GM}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$$

$\boxed{\text{let } r_1 = r_2,\ r_2 = r_1,\ (\text{swapping})}$

$$\Delta v_2 = \sqrt{GM}\left(\sqrt{\frac{1}{r_2}} - \sqrt{\frac{2}{r_2}-\frac{2}{r_1+r_2}}\right)$$

$$= \sqrt{\frac{GM}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right)$$

$\Delta v_1 = 8794.28\ m/s$

$\Delta v_2 = 5643.86\ m/s$

$$E_{\text{tot of sys/spacecraft}} = \frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{2a}$$

where r_1 is the initial orbit
and a is the semi-major of the elliptical 'new' orbit it is moving into.

the reverse is also true.

(b) Rocket equation $\Delta v = v_{exh} \log_e \frac{m_i}{m_f}$,
m_i is initial mass;
m_f is final mass

find total Δ in mass, $\Delta m = m_i - m_f$

- since we are not given the initial mass, we may ~~/fraction~~ express our answer as a percentage change or ratio of initial mass.
- also since the rocket equation is linear (logarithmatic expression) we may sum the individual changes.

$$\Delta v_{tot} = (\Delta v_1 + \Delta v_2) = 2989.1\,m/s \ln\left(\frac{m_i}{m_f}\right)$$

$$e^{\frac{\Delta v_1 + \Delta v_2}{2989.1\,m/s}} = \frac{m_i}{m_f}$$

After manipulation, $1 - \frac{m_f}{m_i} = 1 - e^{-\frac{\Delta v_1 + \Delta v_2}{2989.1\,m/s}}$

$$\frac{\Delta m}{m_i} = \frac{m_i - m_f}{m_i} = 0.992 \text{ of initial mass}$$ #

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